

Study of high-multiplicity three-prong and five-prong τ decays at *BABAR*

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Abstract

We present measurements of the branching fractions of three-prong and five-prong τ decay modes using a sample of 430 million τ lepton pairs, corresponding to an integrated luminosity of 468 fb^{-1} , collected with the *BABAR* detector at the PEP-II asymmetric-energy e^+e^- storage rings at SLAC. The $\tau^- \rightarrow (3\pi)^-\eta\nu_\tau$, $\tau^- \rightarrow (3\pi)^-\omega\nu_\tau$, and $\tau^- \rightarrow \pi^- f_1(1285)\nu_\tau$ branching fractions are presented, as well as a new limit on the branching fraction of the second-class current decay $\tau^- \rightarrow \pi^-\eta'(958)\nu_\tau$. We search for the decay mode $\tau^- \rightarrow K^-\eta'(958)\nu_\tau$ and for five-prong decay modes with kaons, and place the first upper limits on their branching fractions.

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I. INTRODUCTION

Study of the three-prong and five-prong decay modes of the τ lepton, where “prong” refers to the number of charged hadrons (π or K) in the final state, allows one to test the Standard Model and search for evidence of new physics. The large τ lepton data sample collected by the *BABAR* experiment provides an opportunity to perform a comprehensive study of rare, high multiplicity decay modes and to search for forbidden processes.

We present measurements of the $\tau^- \rightarrow (3\pi)^-\eta\nu_\tau$,

$\tau^- \rightarrow (3\pi)^-\omega\nu_\tau$, and $\tau^- \rightarrow \pi^- f_1\nu_\tau$ branching fractions. We use the primary decay modes of the η , $\omega(782)$, and $f_1(1258)$ mesons: $\eta \rightarrow \gamma\gamma$, $\eta \rightarrow \pi^+\pi^-\pi^0$, $\eta \rightarrow 3\pi^0$, $\omega \rightarrow \pi^-\pi^+\pi^0$, $f_1 \rightarrow 2\pi^+2\pi^-$, and $f_1 \rightarrow \pi^+\pi^-\eta$. No other narrow resonances are observed. We measure the branching fractions of the non-resonant decays, where the non-resonant category includes possible contributions from broad resonances. We present a new limit on the branching fractions of the second-class current decay $\tau^- \rightarrow \pi^-\eta'(958)\nu_\tau$, and the first limits on the allowed first-class current decays $\tau^- \rightarrow K^-\eta'(958)\nu_\tau$ and $\tau^- \rightarrow \pi^-\pi^0\eta'(958)\nu_\tau$. Finally, we present the first limits on the branching fractions of five-prong decay modes in which one or more of the charged hadrons is a charged kaon. Note that the branching fractions exclude the contribution of $K_S^0 \rightarrow \pi^+\pi^-$ decays. Throughout this paper, charge conjugation is implied.

This analysis is based on data recorded with the *BABAR* detector at the PEP-II asymmetric-energy e^+e^- storage rings operated at the SLAC National Accelerator Laboratory. With an integrated luminosity (\mathcal{L}) of 424 and 44 fb^{-1} recorded at center-of-mass (CM) energies of

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10.58 GeV and 10.54 GeV, respectively, and an $e^+e^- \rightarrow \tau^+\tau^-$ cross section of $\sigma_{\tau^+\tau^-} = (0.919 \pm 0.003)$ nb [1], the data sample contains 430 million τ pairs.

The *BABAR* detector is described in detail in Ref. [2]. Charged-particle momenta are measured with a five-layer double-sided silicon vertex tracker and a 40-layer drift chamber, both operating in the 1.5 T magnetic field of a superconducting solenoid. Information from a detector of internally reflected Cerenkov light is used in conjunction with specific energy loss measurements from the tracking detectors to identify charged pions and kaons [3]. Photons are reconstructed from energy clusters deposited in a CsI(Tl) electromagnetic calorimeter. Electrons are identified by combining tracking and calorimeter information. An instrumented magnetic flux return is used to identify muons.

The background contamination and selection efficiencies are determined using Monte Carlo simulation. The τ -pair production is simulated with the KK2F event generator [4]. The τ decays, continuum $q\bar{q}$ events ($q = u\bar{d}s\bar{c}$), and final-state radiative effects are modeled with the Tauola [5], JETSET [6], and Photos [7] generators, respectively. Dedicated samples of $\tau^+\tau^-$ events are created using the Tauola or EVTGEN [8] programs, with one of the τ leptons allowed to decay to any mode while the other τ decays to a specific final state. The detector response is simulated with GEANT4 [9]. All Monte Carlo events are processed through a full simulation of the *BABAR* detector and are reconstructed in the same manner as the data.

II. EVENT SELECTION

The τ pair is produced back-to-back in the e^+e^- CM frame. The decay products of the two τ leptons can thus be separated from each other by dividing the event into two hemispheres: the “signal” hemisphere and the “tag” hemisphere. The separation is performed using the event thrust axis [10], which is calculated using all charged particle and photon candidates in the event.

We select events where one hemisphere (tag) contains exactly one track while the other hemisphere (signal) contains exactly three or five tracks with total charge opposite to that of the tag hemisphere. The event is rejected if any pair of oppositely charged tracks is consistent with a photon conversion. The component of the momentum transverse to the beam axis for each of the tracks must be greater than 0.1 GeV/ c in the laboratory frame. All tracks are required to have a point of closest approach to the interaction region less than 1.5 cm in the plane transverse to the beam axis and less than 2.5 cm in the direction along that axis. This requirement eliminates K_S^0 mesons that decay to $\pi^+\pi^-$ at points distant from the e^+e^- collision point.

To reduce backgrounds from non- τ -pair events, we require that the momentum of the charged particle in the

tag hemisphere be less than 4 GeV/ c in the CM frame and that the charged particle be identified as an electron or a muon. The $q\bar{q}$ background is suppressed by requiring that there be at most one energetic ($E > 1$ GeV) electromagnetic calorimeter cluster in the tag hemisphere that is not associated with a track. Additional background suppression is achieved by requiring the magnitude of the event thrust to lie between 0.92 and 0.99.

Neutral pion and eta candidates are reconstructed from two photon candidates, each with energy greater than 30 MeV in the laboratory frame; the invariant mass of the π^0 (η) is required to be between 0.115 (0.35) and 0.150 (0.70) GeV/ c^2 . Neutral pion candidates are reconstructed in the signal hemisphere. If a photon candidate meets the invariant mass requirement with multiple photon candidates, then the neutral pion candidate with invariant mass closest to the nominal π^0 mass [11] is selected. The search for additional pion candidates is repeated using the remaining photon candidates. The residual photon clusters in the signal hemisphere are used to search for $\eta \rightarrow \gamma\gamma$ candidates. In the case of multiple η candidates, the candidate with invariant mass closest to the nominal η mass is selected. We reject events in which the invariant mass M formed from the system of charged particles, π^0 , and η candidates, all in the signal hemisphere, exceeds 1.8 GeV/ c^2 .

The branching fractions are calculated using the expression $\mathcal{B} = N_X/(2N\varepsilon)$ where N_X is the number of candidates after background subtraction, N is the number of τ pairs produced, and ε is the selection efficiency. N is determined from the product of the integrated luminosity and the $e^+e^- \rightarrow \tau^+\tau^-$ cross section. The uncertainty of N is estimated to be 1%. The selection efficiencies are determined from the signal Monte Carlo samples. The uncertainty on the selection efficiencies includes 0.5% per track on the track reconstruction efficiency, as well as particle identification (PID) selection uncertainties. From studies conducted on real and simulated events, the uncertainties on the charged particle identification selectors are estimated to be 1% for electrons, 2.5% for muons, 0.5% for pions, and 1.8% for kaons. The combined electron and muon particle identification uncertainty is estimated to be 1.6% based on the composition of the event samples. The uncertainty on the $\pi^0 \rightarrow \gamma\gamma$ and $\eta \rightarrow \gamma\gamma$ reconstruction efficiency is estimated to be 3% per candidate.

III. RESULTS

We present measurements of τ decays to a system with η , f_1 and ω resonances in Sections A, B, and C, respectively. Decays with these resonances do not account for all three-prong or five-prong τ decay modes, as discussed below, and we present measurements of the τ branching fractions through non-resonant modes in Section D. Section E presents a search for τ decays with an η' (958)

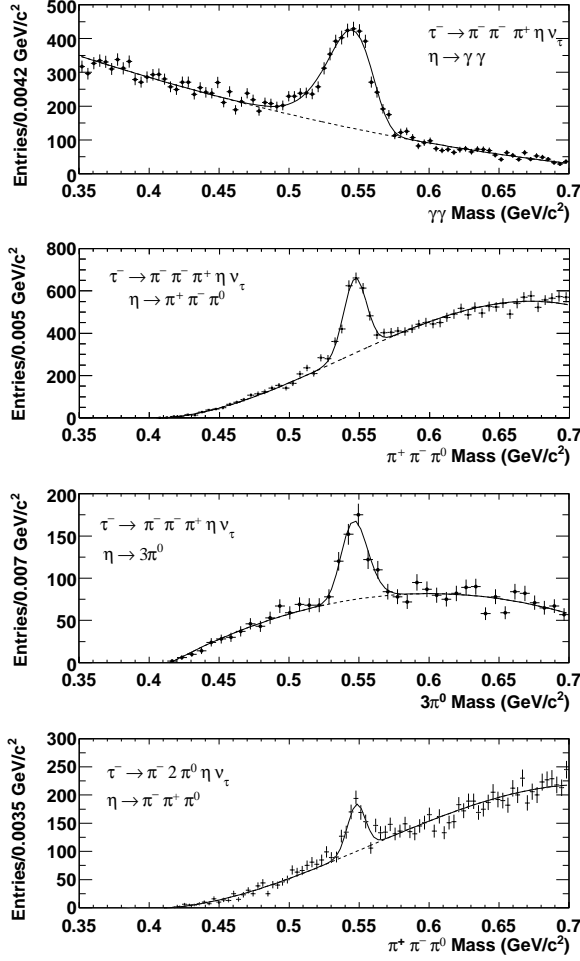


FIG. 1: The $\gamma\gamma$, $\pi^+\pi^-\pi^0$, and $3\pi^0$ invariant mass distributions for $\tau^- \rightarrow 2\pi^-\pi^+\eta\nu_\tau$ decay candidates, and the $\pi^+\pi^-\pi^0$ invariant mass distribution for $\tau^- \rightarrow \pi^-2\pi^0\eta\nu_\tau$ decay candidates, after all selection criteria are applied. The solid lines represent the fit to the η peak and background. The dashed lines show the extrapolation of the background function under the η peak.

meson, while Section F presents a search for decays with either one or two charged kaons.

A. $\tau^- \rightarrow (3\pi)^-\eta\nu_\tau$

The $\tau^- \rightarrow 2\pi^-\pi^+\eta\nu_\tau$ mode is studied in the $\eta \rightarrow \gamma\gamma$, $\eta \rightarrow \pi^+\pi^-\pi^0$, and $\eta \rightarrow 3\pi^0$ final states, while the $\tau^- \rightarrow \pi^-2\pi^0\eta\nu_\tau$ mode is studied in the $\eta \rightarrow \pi^+\pi^-\pi^0$ final state.

The event yields are determined by fitting the η mass peak in the $\gamma\gamma$, $\pi^+\pi^-\pi^0$, and $3\pi^0$ invariant mass distributions (see Fig. 1). The fit uses a Novosibirsk function [12] (a Gaussian distribution with a tail parameter) for the η and a polynomial function for the background.

The Monte Carlo simulation indicates that some of the entries in the η peak are from $e^+e^- \rightarrow q\bar{q}$ events.

Control samples, obtained by reversing the requirement on the invariant mass of the observed decay products ($M > 1.8 \text{ GeV}/c^2$), are used to validate the background estimate. The expected background is corrected by the ratio of data to Monte Carlo events, and the statistical uncertainty of the ratio is included in the background systematic uncertainty. This method of validating the $q\bar{q}$ background estimate is used for all decays and is not mentioned in the later sections.

The reconstruction efficiencies are determined from fits to the signal Monte Carlo samples. The $\tau^- \rightarrow \pi^-2\pi^0\eta\nu_\tau$ sample is generated using a phase-space model for the final-state particles. The $\tau^- \rightarrow 2\pi^-\pi^+\eta\nu_\tau$ sample is composed of $\tau^- \rightarrow \pi^-f_1\nu_\tau$ ($f_1 \rightarrow \pi^+\pi^-\eta$) decays and decays without an intermediate resonance. The $\tau^- \rightarrow 2\pi^-\pi^+\eta\nu_\tau$ (excluding f_1) and $\tau^- \rightarrow \pi^-f_1\nu_\tau$ efficiencies are the same for $\eta \rightarrow \pi^+\pi^-\pi^0$ and $\eta \rightarrow 3\pi^0$ events, whereas a slight difference is observed for $\eta \rightarrow \gamma\gamma$ events and a 2.5% uncertainty is added to the selection efficiency systematic for this mode. In addition, a 4% uncertainty is added to the $\tau^- \rightarrow 2\pi^-\pi^+\eta\nu_\tau$ selection efficiency for the $\eta \rightarrow \pi^+\pi^-\pi^0$ mode to take into account variations observed for different fits.

The three determinations of the $\tau^- \rightarrow 2\pi^-\pi^+\eta\nu_\tau$ branching fraction are found to be in good agreement (see Table I) and we therefore calculate a weighted average. The statistical and systematic uncertainties on the average are obtained by combining the individual uncertainties in quadrature, accounting for correlations between the systematic terms. The weighted average (inclusive of $\tau^- \rightarrow \pi^-f_1\nu_\tau$) is found to be

$$\mathcal{B}(\tau^- \rightarrow 2\pi^-\pi^+\eta\nu_\tau) = (2.25 \pm 0.07 \pm 0.12) \times 10^{-4}.$$

Hereinafter, when two uncertainties are quoted, the first is statistical and the second is systematic. The average branching fraction (exclusive of $\tau^- \rightarrow \pi^-f_1\nu_\tau$) is determined to be $(0.99 \pm 0.09 \pm 0.13) \times 10^{-4}$ and is obtained using the branching fraction (inclusive of $\tau^- \rightarrow \pi^-f_1\nu_\tau$), given above, and subtracting the product branching fraction $\mathcal{B}(\tau^- \rightarrow \pi^-f_1\nu_\tau) \times \mathcal{B}(f_1 \rightarrow \pi^+\pi^-\eta)$ presented in the next section.

The $\tau^- \rightarrow \pi^-2\pi^0\eta\nu_\tau$ branching fraction is found to be

$$\mathcal{B}(\tau^- \rightarrow \pi^-2\pi^0\eta\nu_\tau) = (2.01 \pm 0.34 \pm 0.22) \times 10^{-4}.$$

Naively, the ratio of the $\tau^- \rightarrow 2\pi^-\pi^+\eta\nu_\tau$ to $\tau^- \rightarrow \pi^-2\pi^0\eta\nu_\tau$ branching fractions is expected to be two if the decay is dominated by the $\tau^- \rightarrow \pi^-f_1\nu_\tau$ decay mode (based on the f_1 branching fractions [11]). The data do not support this expectation.

Our previous measurement of the $\tau^- \rightarrow 2\pi^-\pi^+\eta\nu_\tau$ branching fraction $(1.60 \pm 0.05 \pm 0.11) \times 10^{-4}$ [13], which is based on the $\eta \rightarrow \gamma\gamma$ mode only, is superseded by this measurement. The fit used in the previous analysis was performed using a narrower range in the invariant mass distribution, and a background parametrization that is less accurate than in the present study.

The $\tau^- \rightarrow 2\pi^-\pi^+\eta\nu_\tau$ and $\tau^- \rightarrow \pi^-2\pi^0\eta\nu_\tau$ branching fractions are in good agreement with the results from

TABLE I: Results and branching fractions for $\tau^- \rightarrow (3\pi)^- \eta \nu_\tau$ decays.

Decay Mode	$\tau^- \rightarrow 2\pi^- \pi^+ \eta \nu_\tau$ $\eta \rightarrow \gamma\gamma$	$\tau^- \rightarrow 2\pi^- \pi^+ \eta \nu_\tau$ $\eta \rightarrow \pi^+ \pi^- \pi^0$	$\tau^- \rightarrow 2\pi^- \pi^+ \eta \nu_\tau$ $\eta \rightarrow 3\pi^0$	$\tau^- \rightarrow \pi^- 2\pi^0 \eta \nu_\tau$ $\eta \rightarrow \pi^+ \pi^- \pi^0$
Branching fraction (10^{-4})	$2.10 \pm 0.09 \pm 0.13$	$2.37 \pm 0.12 \pm 0.18$	$2.54 \pm 0.27 \pm 0.25$	$2.01 \pm 0.34 \pm 0.22$
Data events	2887 ± 103	1440 ± 68	315 ± 34	381 ± 45
χ^2/NDF	107/76	60/52	31/34	95/75
Selection efficiency	$(3.83 \pm 0.11)\%$	$(2.97 \pm 0.12)\%$	$(0.42 \pm 0.01)\%$	$(0.75 \pm 0.02)\%$
Background events	131 ± 29	65 ± 38	13 ± 7	83 ± 12
Systematic uncertainties (%)				
Tracking efficiency	2.7	3.8	2.7	2.7
π^0 and η PID	3.0	3.0	9.0	9.0
Pion PID	1.5	2.5	1.5	1.5
Lepton-tag PID	1.6	1.6	1.6	1.6
N	1.0	1.0	1.0	1.0
Selection efficiency	3.9	4.0	2.8	2.7
Background	1.0	2.8	2.3	4.0
$\mathcal{B}(\eta \rightarrow \gamma\gamma)$	1.0	-	-	-
$\mathcal{B}(\eta \rightarrow \pi^+ \pi^- \pi^0)$	-	1.8	-	1.8
$\mathcal{B}(\eta \rightarrow 3\pi^0)$	-	-	0.9	-
Total (%)	6.3	7.4	10	11

the CLEO Collaboration, $(2.3 \pm 0.5) \times 10^{-4}$ and $(1.5 \pm 0.5) \times 10^{-4}$, respectively [14]. Li predicts a larger $\tau^- \rightarrow 2\pi^- \pi^+ \eta \nu_\tau$ branching fraction, 2.93×10^{-4} [15].

B. $\tau^- \rightarrow \pi^- f_1 \nu_\tau$

The branching fraction of $\tau^- \rightarrow \pi^- f_1 \nu_\tau$ and the mass of the f_1 meson are measured using the $f_1 \rightarrow 2\pi^+ 2\pi^-$ and $f_1 \rightarrow \pi^+ \pi^- \eta$ decay modes, where the $f_1 \rightarrow \pi^+ \pi^- \eta$ decay is reconstructed using $\eta \rightarrow \gamma\gamma$, $\eta \rightarrow \pi^+ \pi^- \pi^0$, and $\eta \rightarrow 3\pi^0$ events. The criteria used to select the $\tau^- \rightarrow \pi^- f_1 \nu_\tau$ decays for the branching fraction measurement are described earlier. We modify the selection for the mass measurement, dropping the requirement that the track in the tag hemisphere be a lepton and the restriction on the number of photon candidates in the tag hemisphere, to increase the size of the event sample.

The numbers of $\tau^- \rightarrow \pi^- f_1 \nu_\tau$ candidates are determined by fitting the f_1 peak in the $2\pi^+ 2\pi^-$ and $\pi^+ \pi^- \eta$ invariant mass distributions (see Fig. 2). The f_1 lineshape is expected to be a Breit-Wigner distribution, modified by the limited phase space. Previous studies show that the $f_1 \rightarrow a_0^- \pi^+$ ($a_0^-(980) \rightarrow \pi^- \eta$) channel appears to account for all $f_1 \rightarrow \pi^+ \pi^- \eta$ decays [16]. The mass of the $\pi a_0(980)$ system and the τ mass provide a lower and upper limit, respectively, on the f_1 lineshape. We use the four-vectors of the charged pion and $a_0(980)$ from the EVTGEN generator to determine the simulated f_1 lineshape and find it to be a close approximation to the Breit-Wigner expectation. The f_1 peak is fit using this

lineshape convolved with a Gaussian distribution to take into account the effects of the detector resolution. The results of the fits are presented in Table II. There is no evidence for peaking background from $q\bar{q}$ events or other τ decays.

The product of the $\tau^- \rightarrow \pi^- f_1 \nu_\tau$ and $f_1 \rightarrow 2\pi^+ 2\pi^-$ branching fractions, and the product of the $\tau^- \rightarrow \pi^- f_1 \nu_\tau$ and $f_1 \rightarrow \pi^+ \pi^- \eta$ branching fractions, are measured to be

$$\begin{aligned}
\mathcal{B}(\tau^- \rightarrow \pi^- f_1 \nu_\tau) \mathcal{B}(f_1 \rightarrow 2\pi^+ 2\pi^-) &= (5.20 \pm 0.31 \pm 0.37) \times 10^{-5}, \\
\mathcal{B}(\tau^- \rightarrow \pi^- f_1 \nu_\tau) \mathcal{B}(f_1 \rightarrow \pi^+ \pi^- \eta) &= (1.26 \pm 0.06 \pm 0.06) \times 10^{-4},
\end{aligned}$$

respectively, where the second result is the weighted average of the three η modes. The $\mathcal{B}(\tau^- \rightarrow \pi^- f_1 \nu_\tau)$ branching fraction is determined to be $(4.73 \pm 0.28 \pm 0.45) \times 10^{-4}$ and $(3.60 \pm 0.18 \pm 0.23) \times 10^{-4}$, as obtained by dividing the product branching fractions by $\mathcal{B}(f_1 \rightarrow 2\pi^+ 2\pi^-) = 0.110^{+0.007}_{-0.006}$ and $\mathcal{B}(f_1 \rightarrow \pi^+ \pi^- \eta) = 0.349^{+0.013}_{-0.015}$ [17], respectively.

Our two measured values for the $\tau^- \rightarrow \pi^- f_1 \nu_\tau$ branching fraction are consistent with each other to within two standard deviations of the combined statistical and systematic uncertainties. The ratio of the product branching fractions is used to determine the ratio of the $f_1 \rightarrow 2\pi^+ 2\pi^-$ and $f_1 \rightarrow \pi^+ \pi^- \eta$ branching fractions as

$$\frac{\mathcal{B}(f_1 \rightarrow 2\pi^+ 2\pi^-)}{\mathcal{B}(f_1 \rightarrow \pi\pi\eta)} = 0.28 \pm 0.02 \pm 0.02,$$

TABLE II: Results and branching fractions for $\tau^- \rightarrow \pi^- f_1 \nu_\tau$ decays.

Decay Mode	$f_1 \rightarrow 2\pi^+ 2\pi^-$	$f_1 \rightarrow \pi^+ \pi^- \eta$ $\eta \rightarrow \gamma\gamma$	$f_1 \rightarrow \pi^+ \pi^- \eta$ $\eta \rightarrow \pi^+ \pi^- \pi^0$	$f_1 \rightarrow \pi^+ \pi^- \eta$ $\eta \rightarrow 3\pi^0$
Branching fractions (10^{-4})				
$\mathcal{B}(\tau^- \rightarrow \pi^- f_1 \nu_\tau) \mathcal{B}(f_1 \rightarrow 2\pi^+ 2\pi^-)$	$0.520 \pm 0.031 \pm 0.037$			
$\mathcal{B}(\tau^- \rightarrow \pi^- f_1 \nu_\tau) \mathcal{B}(f_1 \rightarrow \pi^+ \pi^- \eta)$		$1.25 \pm 0.08 \pm 0.07$	$1.26 \pm 0.11 \pm 0.08$	$1.33 \pm 0.39 \pm 0.20$
Data events	3722 ± 222	1605 ± 94	731 ± 62	197 ± 59
χ^2/NDF	77/62	50/43	61/55	39/43
Selection efficiency	$(8.3 \pm 0.1)\%$	$(3.75 \pm 0.04)\%$	$(2.97 \pm 0.05)\%$	$(0.53 \pm 0.06)\%$
Systematic uncertainties (%)				
Tracking efficiency	3.8	2.7	3.8	2.7
π^0 and η PID	-	3.0	3.0	9.0
Pion PID	2.5	1.5	2.5	1.5
Lepton-tag PID	1.6	1.6	1.6	1.6
N	1.0	1.0	1.0	1.0
Selection efficiency	0.6	1.1	1.6	11
Fit model	5.0	2.7	-	-
$\mathcal{B}(\eta \rightarrow \gamma\gamma)$	-	0.7	-	-
$\mathcal{B}(\eta \rightarrow \pi^+ \pi^- \pi^0)$	-	-	1.2	-
$\mathcal{B}(\eta \rightarrow 3\pi^0)$	-	-	-	0.9
Total (%)	7.0	5.6	6.1	15

TABLE III: Results of fits for the mass of the f_1 resonance in $\tau^- \rightarrow \pi^- f_1 \nu_\tau$ decays. The errors are statistical.

Decay Mode	Monte Carlo (generator - fit) (GeV/ c^2)	Data (fit) (GeV/ c^2)	Data (corrected) (GeV/ c^2)
$f_1 \rightarrow 2\pi^+ 2\pi^-$	0.00074 ± 0.00008	1.28031 ± 0.00067	1.28105 ± 0.00067
$f_1 \rightarrow \pi^+ \pi^- \eta$			
$\eta \rightarrow \gamma\gamma$	0.00292 ± 0.00040	1.27775 ± 0.00045	1.28067 ± 0.00060
$\eta \rightarrow \pi^+ \pi^- \pi^0$	0.00018 ± 0.00020	1.27787 ± 0.00080	1.27805 ± 0.00082
$\eta \rightarrow 3\pi^0$	0.00347 ± 0.00033	1.28036 ± 0.00335	1.28383 ± 0.00337

where $\mathcal{B}(f_1 \rightarrow \pi\pi\eta) = 1.5 \times \mathcal{B}(f_1 \rightarrow \pi^+ \pi^- \eta)$ based on isospin symmetry. This agrees with average value of 0.41 ± 0.14 quoted by the Particle Data Group [11] but disagrees with their fit value of 0.63 ± 0.06 [11].

The systematic uncertainties of the branching fractions are listed in Table II. We observe that the number of events in the f_1 peak in the $f_1 \rightarrow 2\pi^+ 2\pi^-$ sample varies by 5% for different background shapes. This variation is included as a systematic uncertainty. We also observe that the selection efficiency obtained from the Monte Carlo simulation exhibits a slight dependence on whether the f_1 decays via the $f_1 \rightarrow a_0^- \pi^+$ or the $f_1 \rightarrow \pi^+ \pi^- \eta$ mode, and the variation is included as a systematic uncertainty (listed under “Fit model” in Table II).

The $\tau^- \rightarrow \pi^- f_1 \nu_\tau$ branching fraction using the $f_1 \rightarrow$

$2\pi^+ 2\pi^-$ mode is consistent with the previous *BABAR* measurement, which is also based on the $f_1 \rightarrow 2\pi^+ 2\pi^-$ mode [18]. CLEO published a branching fraction of $(5.8^{+1.4}_{-1.3} \pm 1.8) \times 10^{-4}$ [19] and Li predicts a branching fraction of 2.9×10^{-4} [20].

The f_1 mass is determined by fitting the peak with a non-relativistic Breit-Wigner function, which was used in previous measurements of the f_1 mass [11]. As a cross check, we use the energy-momentum four-vectors from the generator Monte Carlo simulation, and we find the fitted mass value to be consistent with the input mass value.

We fit the invariant mass distribution in the fully reconstructed Monte Carlo samples to determine whether

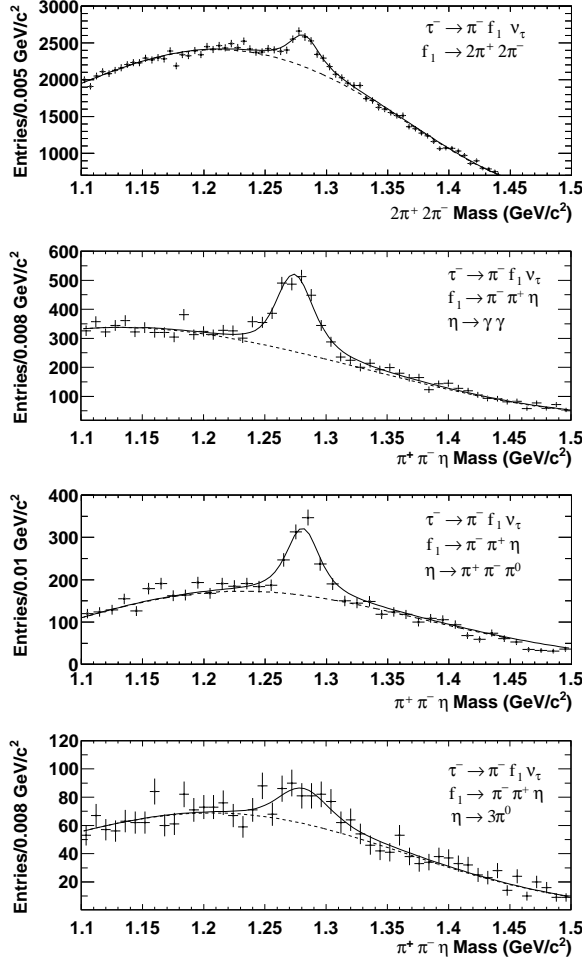


FIG. 2: The $2\pi^+ 2\pi^-$ and $\pi^+ \pi^- \eta$ invariant mass distributions for $\tau^- \rightarrow 2\pi^- \pi^+ \eta \nu_\tau$ decay candidates after all selection criteria are applied. The lower three plots are for the $\eta \rightarrow \gamma\gamma$, $\eta \rightarrow \pi^+ \pi^- \pi^0$, and $\eta \rightarrow 3\pi^0$ decays. The solid lines represent the fit to the $f_1(1285)$ peak and background. The dashed lines show the extrapolation of the background function under the f_1 peak.

the result of the fit differs from the input mass of the Monte Carlo generator. The difference is used to correct the value of the invariant mass of each channel obtained from the fit and the uncertainty in the difference is included as a systematic error.

Table III and Fig. 3 show the results of the fits to the data. The last column of the table presents the mass after the application of the reconstruction correction factor. The average of these results is $M_{f_1} = (1.28025 \pm 0.00039) \text{ GeV}/c^2$, where the error is statistical.

Previous *BABAR* analyses have measured the invariant mass of resonances to be approximately $1 \text{ MeV}/c^2$ less than the values of Ref. [11]. This shift is observed in the measurement of the mass of the f_1 meson [21] and the τ lepton [22]. The shift is attributed to the absolute energy and momentum calibration of the detector.

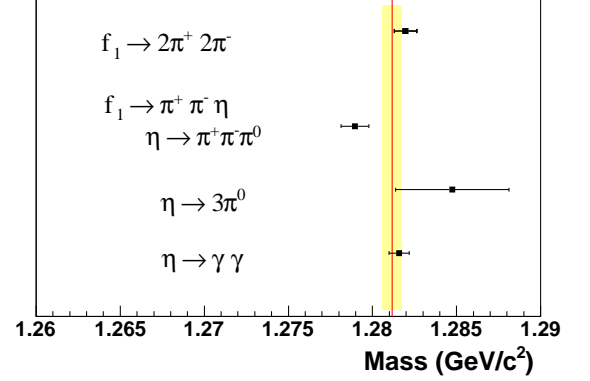


FIG. 3: Compilation of our measurements of the f_1 mass. The solid line is the weighted average and the shaded area is the one-standard-deviation region.

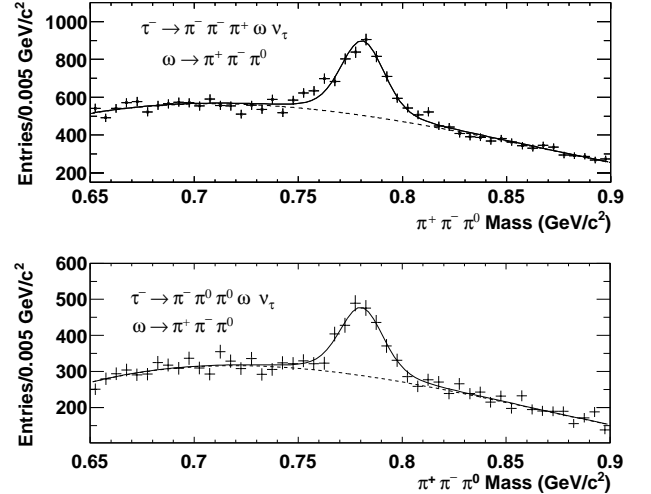


FIG. 4: The fits to the ω peak in the $\pi^+ \pi^- \pi^0$ invariant mass distributions for $\tau^- \rightarrow 2\pi^- \pi^+ \omega \nu_\tau$ and $\tau^- \rightarrow \pi^- 2\pi^0 \omega \nu_\tau$ decay candidates after all selection criteria are applied. The solid lines represent the fit to the ω peak and background. The dashed lines show the extrapolation of the background function under the ω peak.

We measure the calibration correction factor by fitting the η , ω , η' , D^0 and D^{*-} states using data samples that have one track in the tag hemisphere and either three or five tracks in the signal hemisphere. No other selection criteria are applied. The peak masses are found to be lower than the known values by $(-0.91 \pm 0.10) \text{ MeV}/c^2$ and the values are independent of the mass of the resonance. The calibration correction factor is applied to the invariant mass and its error is included in the systematic uncertainty.

We determine the mass of the $f_1(1258)$ meson to be

$$M_{f_1} = (1.28116 \pm 0.00039 \pm 0.00045) \text{ GeV}/c^2.$$

The systematic uncertainty includes the reconstruction uncertainty and the calibration uncertainty. This result is in good agreement with the value $(1.2818 \pm 0.0006) \text{ GeV}/c^2$ in Ref. [11].

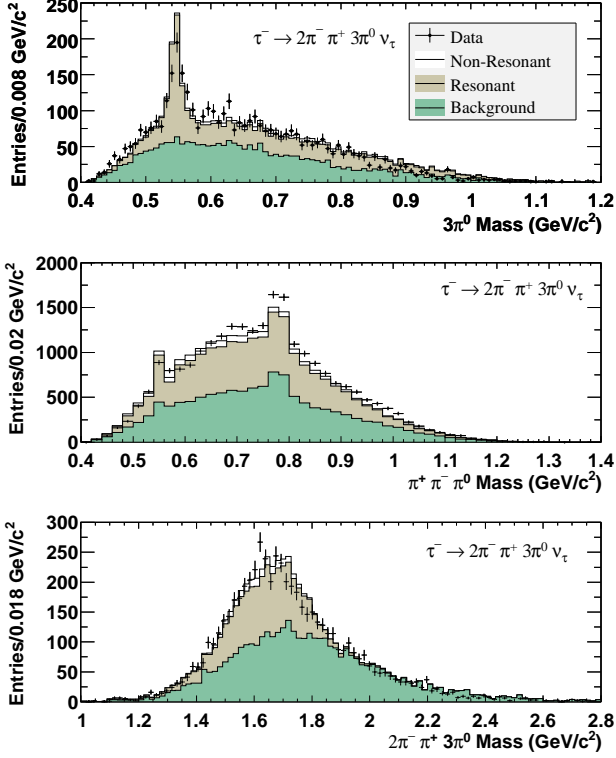


FIG. 5: The $3\pi^0$, $\pi^+\pi^-\pi^0$, and $2\pi^-\pi^+3\pi^0$ invariant mass distributions in $\tau^- \rightarrow 2\pi^-\pi^+3\pi^0\nu_\tau$ decay candidates. The predictions of the Monte Carlo simulation are shown for the resonant and non-resonant τ decays, and the background from other τ decays and $q\bar{q}$ events. The resonant decays include decays with the correct topology and a resonance (η , f_1 or ω) in the final state.

C. $\tau^- \rightarrow (3\pi)^-\omega\nu_\tau$

We measure the $\tau^- \rightarrow 2\pi^-\pi^+\omega\nu_\tau$ and $\tau^- \rightarrow \pi^-2\pi^0\omega\nu_\tau$ branching fractions. The number of events is determined by fitting the ω peak in the $\pi^+\pi^-\pi^0$ invariant mass distributions (see Fig. 4) with a Breit-Wigner distribution, which is convolved with a Gaussian distribution to take into account the detector resolution. The resolution parameter of the Gaussian distribution is determined using a data control sample consisting of $q\bar{q}$ events, and is fixed in the fit. A polynomial function is used to fit the background. The results are presented in Table IV.

Approximately 10% of the events in the $\tau^- \rightarrow 2\pi^-\pi^+\omega\nu_\tau$ channel are backgrounds from other τ decays (primarily $\tau^- \rightarrow \pi^-\pi^0\omega\nu_\tau$ decays) and $e^+e^- \rightarrow q\bar{q}$

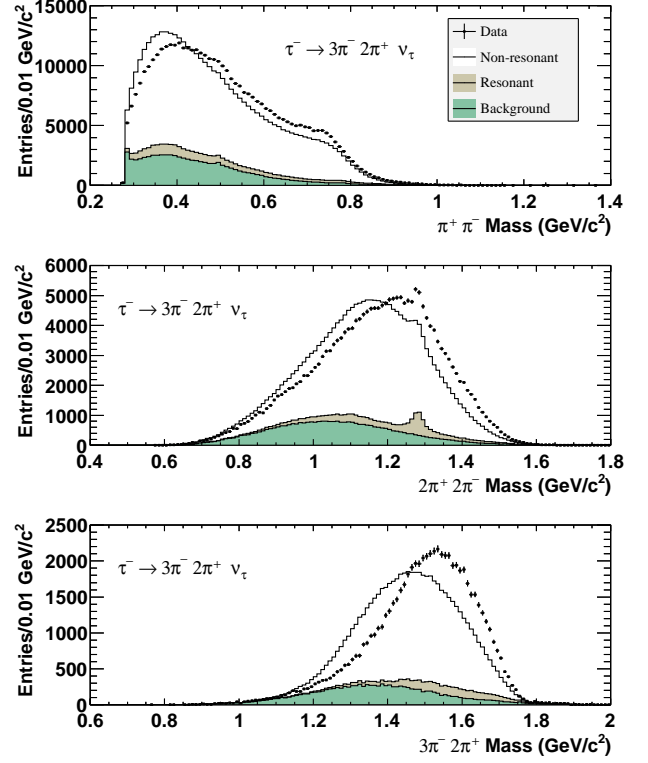


FIG. 6: The $\pi^+\pi^-$, $2\pi^+2\pi^-$, and $3\pi^-2\pi^+$ invariant mass distributions in $\tau^- \rightarrow 3\pi^-2\pi^+\nu_\tau$ decays. The predictions of the Monte Carlo simulation are shown for the resonant and non-resonant τ decays, and the background from other τ decays and $q\bar{q}$ events. The resonant decays include decays with the correct topology and a resonance (η , f_1 or ω) in the final state. The non-resonant decays are generated using $\tau^- \rightarrow a_1^-\nu_\tau$ events. The differences between the data and Monte Carlo predictions are discussed in the text.

events. The backgrounds are subtracted before calculating the branching fraction.

The $\tau^- \rightarrow \pi^-2\pi^0\omega\nu_\tau$ sample has substantial contributions from $\tau^- \rightarrow \pi^-\omega\nu_\tau$ and $\tau^- \rightarrow \pi^-\pi^0\omega\nu_\tau$ decays. The background is estimated with the Monte Carlo simulation and verified using data and simulation control samples. The control samples follow the nominal selection criteria but select one or two π^0 instead of three π^0 mesons.

The branching fractions are found to be

$$\begin{aligned} \mathcal{B}(\tau^- \rightarrow 2\pi^-\pi^+\omega\nu_\tau) &= (8.4 \pm 0.4 \pm 0.6) \times 10^{-5}, \\ \mathcal{B}(\tau^- \rightarrow \pi^-2\pi^0\omega\nu_\tau) &= (7.3 \pm 1.2 \pm 1.2) \times 10^{-5}. \end{aligned}$$

The systematic uncertainties are listed in Table IV.

The $\tau^- \rightarrow 2\pi^-\pi^+\omega\nu_\tau$ and $\tau^- \rightarrow \pi^-2\pi^0\omega\nu_\tau$ branching fractions are consistent with the results from CLEO, $(1.2 \pm 0.2 \pm 0.1) \times 10^{-4}$ and $(1.4 \pm 0.4 \pm 0.3) \times 10^{-4}$, respectively [14]. Gao and Li suggest that this mode is dominated by the $(\pi\rho\omega)$ state and predict a branching fraction in the range of $1.8 - 2.1 \times 10^{-4}$ with the two modes ($\tau^- \rightarrow 2\pi^-\pi^+\omega\nu_\tau$ and $\tau^- \rightarrow \pi^-2\pi^0\omega\nu_\tau$) having

TABLE IV: Results and branching fractions for $\tau^- \rightarrow (3\pi)^- \omega \nu_\tau$ decays.

Decay Mode	$\tau^- \rightarrow 2\pi^- \pi^+ \omega \nu_\tau$ $\omega \rightarrow \pi^- \pi^+ \pi^0$	$\tau^- \rightarrow \pi^- 2\pi^0 \omega \nu_\tau$ $\omega \rightarrow \pi^- \pi^+ \pi^0$
Branching fractions (10^{-5})	$8.4 \pm 0.4 \pm 0.6$	$7.3 \pm 1.2 \pm 1.2$
Data events	2372 ± 94	1135 ± 70
χ^2/NDF	55/44	42/44
Selection efficiency	$(3.27 \pm 0.03)\%$	$(0.75 \pm 0.01)\%$
Background	257 ± 71	709 ± 59
Systematic uncertainties (%)		
Tracking efficiency	3.8	2.7
π^0 and η PID	3.0	9.0
Pion PID	2.5	1.5
Lepton-tag PID	1.6	1.6
N	1.0	1.0
Selection efficiency	0.8	1.8
Background	3.4	14
$\mathcal{B}(\omega \rightarrow \pi^- \pi^+ \pi^0)$	0.8	0.8
Total (%)	6.8	17

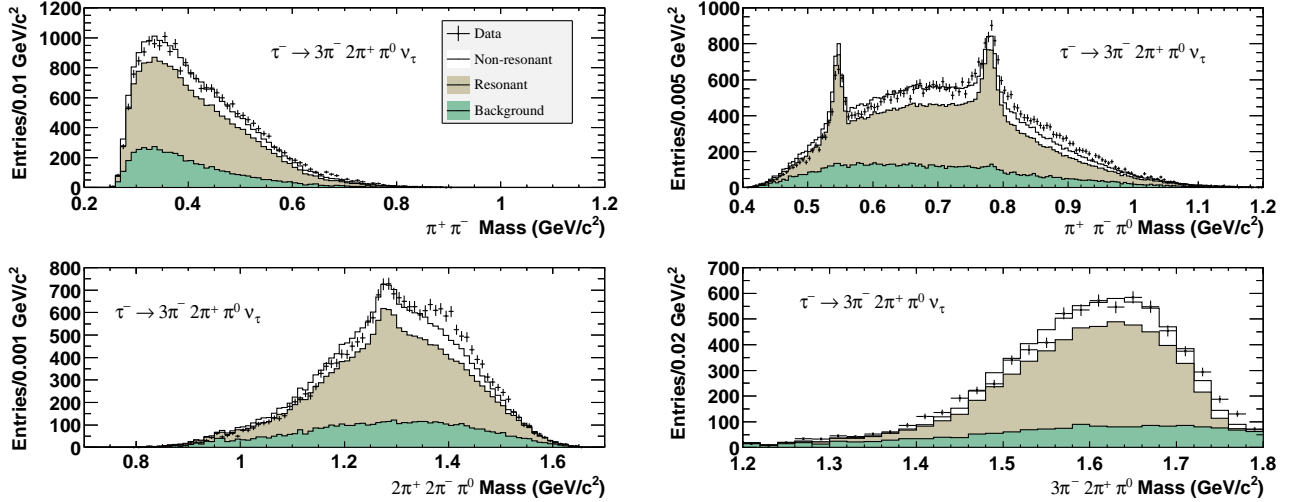


FIG. 7: The $\pi^+ \pi^-$, $\pi^+ \pi^- \pi^0$, $2\pi^- 2\pi^+ \pi^0$, and $3\pi^- 2\pi^+ \pi^0$ invariant mass distributions in $\tau^- \rightarrow 3\pi^- 2\pi^+ \pi^0 \nu_\tau$ decays. The predictions of the Monte Carlo simulation are shown for the resonant and non-resonant τ decays, and the background from other τ decays and $q\bar{q}$ events. The resonant decays include decays with correct topology and a resonance (η , f_1 or ω) in the final state.

the same value [23]. The result measured in this work is approximately 50% of the predicted rate but the ratio of the two branching fractions is consistent with unity.

D. Non-resonant decay modes

The resonant modes, involving η , ω and f_1 mesons, do not account for all of the observed decays, as discussed below. We consider the excess in the observed

decays to be from “non-resonant” modes. We make no attempt to identify the contribution of resonances with larger widths ($> 100 \text{ MeV}/c^2$) as the nature of these resonances is complex and their lineshapes will be modified by the limited phase space in the τ decay. The Monte Carlo simulation describes the final states using a phase-space model for the final-state particles. The only exception is the $\tau^- \rightarrow 3\pi^- 2\pi^+ \nu_\tau$ mode, which Tauola models using $\tau^- \rightarrow a_1^- \nu_\tau$ decays [24].

We measure the branching fractions of the non-

TABLE V: Results and branching fractions for $\tau^- \rightarrow 2\pi^-\pi^+3\pi^0\nu_\tau$, $\tau^- \rightarrow 3\pi^-2\pi^+\nu_\tau$, and $\tau^- \rightarrow 3\pi^-2\pi^+\pi^0\nu_\tau$ non-resonant decays.

Decay Mode	$\tau^- \rightarrow 2\pi^-\pi^+3\pi^0\nu_\tau$	$\tau^- \rightarrow 3\pi^-2\pi^+\nu_\tau$	$\tau^- \rightarrow 3\pi^-2\pi^+\pi^0\nu_\tau$
Branching fractions (10^{-4})	$0.10 \pm 0.08 \pm 0.30$	$7.68 \pm 0.04 \pm 0.40$	$0.36 \pm 0.03 \pm 0.09$
Data events	4094 ± 64	68985 ± 263	7296 ± 85
Efficiency	$(0.88 \pm 0.01)\%$	$(7.98 \pm 0.02)\%$	$(3.71 \pm 0.03)\%$
Background			
Resonant	1763 ± 222	4441 ± 370	4458 ± 244
Other τ decays	1681 ± 44	10621 ± 719	1315 ± 100
$q\bar{q}$	573 ± 50	1171 ± 205	359 ± 37
Total Background	4017 ± 232	16233 ± 835	6132 ± 267
Systematic uncertainties (%)			
Tracking efficiency		3.8	3.8
π^0 and η PID		-	3.0
Pion PID		2.5	2.5
Lepton-tag PID		1.6	1.6
$\mathcal{L} \sigma_{\tau^+\tau^-}$		1.0	1.0
Selection efficiency		0.2	0.9
Background		1.6	22.9
Total (%)		5.2	23.7

resonant $\tau^- \rightarrow 2\pi^-\pi^+3\pi^0\nu_\tau$, $\tau^- \rightarrow 3\pi^-2\pi^+\nu_\tau$, and $\tau^- \rightarrow 3\pi^-2\pi^+\pi^0\nu_\tau$ decays. The numbers of candidates are given by the numbers of events found in the data after subtracting the resonant contributions and the background from other τ decays and $q\bar{q}$ events (see Table V).

The invariant mass plots in Fig. 5 show that the resonant decays dominate the $\tau^- \rightarrow 2\pi^-\pi^+3\pi^0\nu_\tau$ mode. The background is primarily from $\tau^- \rightarrow \pi^-\pi^0\omega\nu_\tau$ and $q\bar{q}$ events. The branching fraction of the non-resonant $\tau^- \rightarrow 2\pi^-\pi^+3\pi^0\nu_\tau$ channel is determined to be $(1.0 \pm 0.8 \pm 3.0) \times 10^{-5}$. The systematic uncertainty on the branching fraction is dominated by the uncertainty in the background, which includes the Monte Carlo statistical uncertainty and the τ branching fraction uncertainties. We do not list the fractional systematic uncertainties for this mode in Table V due to the smallness of the branching fraction. The branching fraction is consistent with zero and we set a limit of

$$\mathcal{B}(\tau^- \rightarrow 2\pi^-\pi^+3\pi^0\nu_\tau) < 5.8 \times 10^{-5}$$

at the 90% confidence level.

We also determine the inclusive $\tau^- \rightarrow 2\pi^-\pi^+3\pi^0\nu_\tau$ branching fraction, given by the sum of the resonant and non-resonant terms. We find the result $(2.07 \pm 0.18 \pm 0.37) \times 10^{-4}$, where the systematic uncertainty accounts for correlations between the systematic uncertainties of the individual modes.

The $\tau^- \rightarrow 3\pi^-2\pi^+\nu_\tau$ decay has only a small contribution from resonant decays (see Fig. 6). The branching fraction of the non-resonant $\tau^- \rightarrow 3\pi^-2\pi^+\nu_\tau$ decay is

determined to be

$$\mathcal{B}(\tau^- \rightarrow 3\pi^-2\pi^+\nu_\tau) = (7.68 \pm 0.04 \pm 0.40) \times 10^{-4}.$$

The $\tau^- \rightarrow 2\pi^-\pi^+\omega\nu_\tau$ ($\omega \rightarrow \pi^-\pi^+\gamma$) decay is considered as a resonant background and is not included in the non-resonant branching fraction. Although the modeling of the $3\pi^-2\pi^+$ invariant mass distribution is deficient, the selection efficiency remains the same if the Monte Carlo is re-weighted to resemble the data distribution. The decay model represents a significant improvement compared to a phase-space model, in which the ρ meson, observed in the $\pi^+\pi^-$ invariant mass spectrum, is not included. Further tuning of the model is required, which is outside the scope of the present study. The background from the $q\bar{q}$ events is validated by comparing the numbers of data and Monte Carlo events in the region above the τ lepton mass. The inclusive $\tau^- \rightarrow 3\pi^-2\pi^+\nu_\tau$ branching fraction is $(8.33 \pm 0.04 \pm 0.43) \times 10^{-4}$ and is obtained by adding the non-resonant branching fraction with the resonant branching fraction for the $\tau^- \rightarrow \pi^-f_1\nu_\tau$ via $f_1 \rightarrow 2\pi^+2\pi^-$ decay. The branching fraction of the $\tau^- \rightarrow 3h^-2h^+\nu_\tau$ decay (where h^- is either a π^- or K^-) was measured to be $(8.56 \pm 0.05 \pm 0.42) \times 10^{-4}$ in a previous *BABAR* analysis [18] using a smaller data sample, which did not use charged particle identification and in which the branching fraction included the contribution of the $\tau^- \rightarrow 2\pi^-\pi^+\omega\nu_\tau$ decay.

$\tau^- \rightarrow 3\pi^-2\pi^+\pi^0\nu_\tau$ decays are dominated by the resonant modes (see Fig. 7). We determine the branching fraction of the non-resonant $\tau^- \rightarrow 3\pi^-2\pi^+\pi^0\nu_\tau$ decay

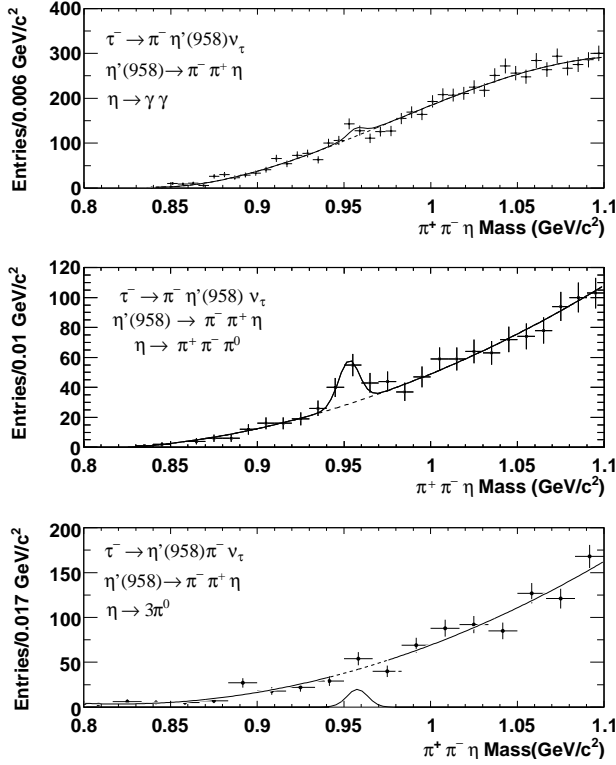


FIG. 8: The $\pi^+\pi^-\eta$ invariant mass in $\tau^- \rightarrow 2\pi^-\pi^+\eta\nu_\tau$ decay candidates for the $\eta \rightarrow \gamma\gamma$, $\eta \rightarrow \pi^+\pi^-\pi^0$, and $\eta \rightarrow 3\pi^0$ decay modes after all selection criteria are applied. The fit to the η' peak (in the top two plots) is represented by the solid line. The number of η' candidates in the $\eta \rightarrow 3\pi^0$ channel is determined by counting the number of events in a single bin centered on the η' mass and subtracting the combinatorial events. The number of combinatorial events is determined by fitting the data (the solid line in the bottom plot), excluding the data point near the η' peak. The peak in this plot indicates the expected location and width of an η' signal.

mode to be

$$\mathcal{B}(\tau^- \rightarrow 3\pi^-2\pi^+\pi^0\nu_\tau) = (3.6 \pm 0.3 \pm 0.9) \times 10^{-5}.$$

The systematic uncertainty on the non-resonant $\tau^- \rightarrow 3\pi^-2\pi^+\pi^0\nu_\tau$ branching fraction is dominated by the large uncertainty in the background (see Table V). Although the invariant mass distributions of the resonant modes in the Monte Carlo simulation are corrected to provide better agreement with the data, the corrections make little difference to the final branching fraction result. The other τ decays and the $q\bar{q}$ events contribute to a lesser extent; their contribution to the uncertainty of the background is very small.

The $\tau^- \rightarrow 3\pi^-2\pi^+\pi^0\nu_\tau$ (including ω and excluding η) branching fraction is $(1.11 \pm 0.04 \pm 0.09) \times 10^{-4}$ and is obtained by adding the non-resonant branching fraction and the resonant branching fraction attributed to the $\tau^- \rightarrow 2\pi^-\pi^+\omega\nu_\tau$ via $\omega \rightarrow \pi^-\pi^+\pi^0$ decay. In addition, the inclusive $\tau^- \rightarrow 3\pi^-2\pi^+\pi^0\nu_\tau$ branching frac-

tion is $(1.65 \pm 0.05 \pm 0.09) \times 10^{-4}$ and is similarly obtained by adding the non-resonant branching fraction and the resonant branching fractions attributed to the $\tau^- \rightarrow 2\pi^-\pi^+\eta\nu_\tau$ via $\eta \rightarrow \pi^+\pi^-\pi^0$ and $\tau^- \rightarrow 2\pi^-\pi^+\omega\nu_\tau$ via $\omega \rightarrow \pi^-\pi^+\pi^0$ decays.

The $\tau^- \rightarrow 3\pi^-2\pi^+\pi^0\nu_\tau$ (including ω and excluding η) branching fraction can be compared with isospin model predictions [25, 26]. There are three τ decay modes with six pions in the final state: $\tau^- \rightarrow 2\pi^-\pi^+3\pi^0\nu_\tau$, $\tau^- \rightarrow 3\pi^-2\pi^+\pi^0\nu_\tau$, and $\tau^- \rightarrow \pi^-5\pi^0\nu_\tau$ (there are no measurements of the $\tau^- \rightarrow \pi^-5\pi^0\nu_\tau$ decay mode). There are four possible isospin states for six pion decays: $(4\pi\rho)$, (3ρ) , $(3\pi\omega)$, and $(\pi\rho\omega)$. The relative rates of the decays can be used to identify the dominant isospin states. The approximate equality of the $\tau^- \rightarrow 2\pi^-\pi^+3\pi^0\nu_\tau$ and $\tau^- \rightarrow 3\pi^-2\pi^+\pi^0\nu_\tau$ branching fractions suggest that the $(4\pi\rho)$ and $(\pi\rho\omega)$ modes should dominate. The limited phase space imposed by the τ mass suppresses the higher mass states and as a result we do not observe evidence of the ρ meson in these decays.

E. Search for decays involving η' (958) decays

We also search for the $\tau^- \rightarrow \pi^-\pi^0\eta'(958)\nu_\tau$, $\tau^- \rightarrow K^-\eta'(958)\nu_\tau$, and $\tau^- \rightarrow \pi^-\eta'(958)\nu_\tau$ decays, where $\eta' \rightarrow \pi^-\pi^+\eta$. The first two decays are allowed first-class-current decays whereas the last decay is a second-class-current decay, with a rate that would be zero in the limit of perfect isospin symmetry.

The numbers of η' candidates in the data and background Monte Carlo samples are given in Table VI. For the $\tau^- \rightarrow \pi^-\pi^0\eta'(958)\nu_\tau$ via $\eta \rightarrow \gamma\gamma$ and the $\tau^- \rightarrow \pi^-\eta'(958)\nu_\tau$ via $\eta \rightarrow \gamma\gamma$ and $\eta \rightarrow \pi^+\pi^-\pi^0$ modes, we measure the number of η' candidates by fitting the peak with a Gaussian function and the combinatoric background with a polynomial function. The number of η' candidates in the other channels is determined by counting the number of events in a single bin centered on the η' mass and subtracting the combinatorial events. The level of the combinatorial background is estimated by fitting the mass spectrum or from the average level of the sideband region around the η' peak.

The $\pi^+\pi^-\eta$ invariant mass distributions for the $\tau^- \rightarrow \pi^-\eta'(958)\nu_\tau$ candidate decays are shown in Fig. 8. Although we see an η' peak in the $\eta \rightarrow \pi^+\pi^-\pi^0$ channel, we find that it can be fully accounted for by $q\bar{q}$ events. We do not show the invariant mass distributions for the $\tau^- \rightarrow \pi^-\pi^0\eta'(958)\nu_\tau$ and $\tau^- \rightarrow K^-\eta'(958)\nu_\tau$ decays. The analysis of these decay modes uses only the $\eta \rightarrow \gamma\gamma$ and $\eta \rightarrow \pi^+\pi^-\pi^0$ channels. The $\eta \rightarrow 3\pi^0$ channel was not considered due to the limited size of the samples.

The results for the three decay modes are given in Table VI. The selection efficiencies are determined with the signal Monte Carlo samples. The backgrounds from η' mesons are attributed to $e^+e^- \rightarrow q\bar{q}$ events and estimated using the Monte Carlo samples. The background estimations are validated by comparing the prediction of

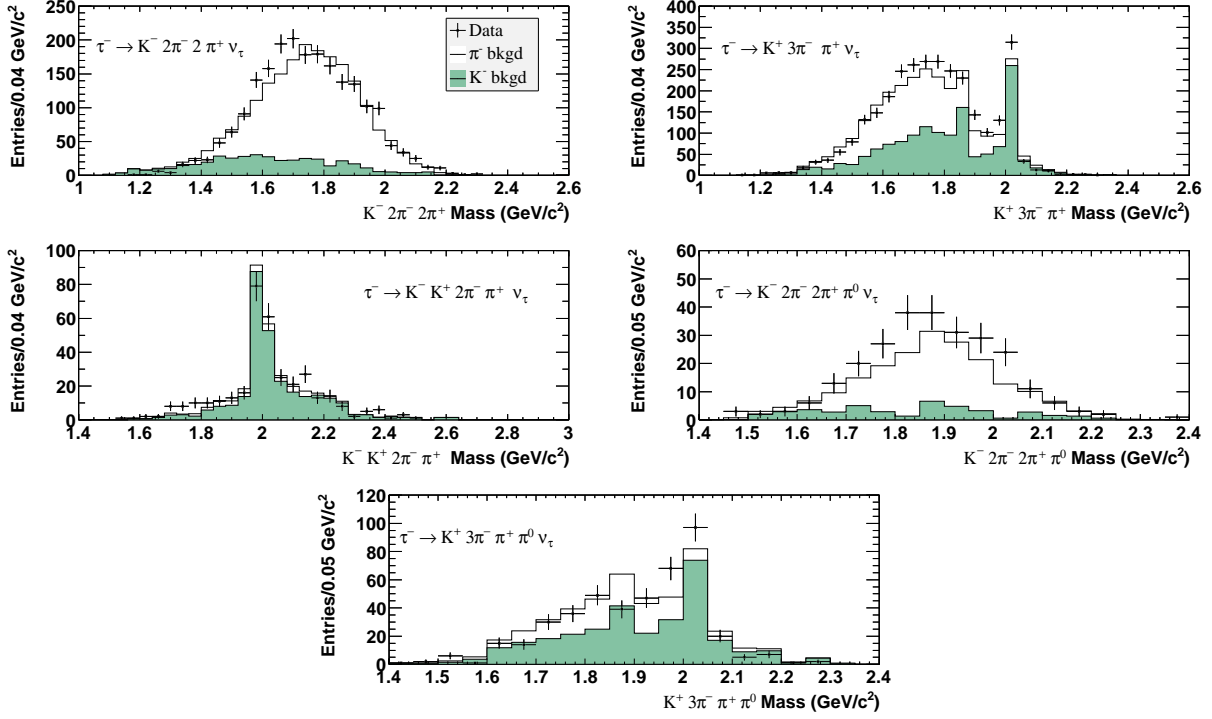


FIG. 9: The $K^-2\pi^-2\pi^+$, $K^+3\pi^-\pi^+$, $K^-K^+2\pi^-\pi^+$, $K^-2\pi^-2\pi^+\pi^0$, and $K^+3\pi^-\pi^+\pi^0$ invariant mass distributions in the data sample after all selection criteria are applied. The unshaded histogram represents τ decays in which a charged pion is misidentified as a charged kaon, and the shaded histograms are primarily from $q\bar{q}$ events in which there is a charged kaon in the final state. The Monte Carlo simulation does not include signal decays.

the Monte Carlo simulation with data for events where the invariant mass of all the observed final-state particles is greater than the τ mass.

We find no evidence for $\tau^- \rightarrow \pi^-\pi^0\eta'(958)\nu_\tau$, $\tau^- \rightarrow K^-\eta'(958)\nu_\tau$, or $\tau^- \rightarrow \pi^-\eta'(958)\nu_\tau$ decays (see Table VI) and place the following upper limits on the branching fractions at the 90% confidence level:

$$\begin{aligned} \mathcal{B}(\tau^- \rightarrow \pi^-\pi^0\eta'(958)\nu_\tau) &< 1.2 \times 10^{-5}, \\ \mathcal{B}(\tau^- \rightarrow K^-\eta'(958)\nu_\tau) &< 2.4 \times 10^{-6}, \\ \mathcal{B}(\tau^- \rightarrow \pi^-\eta'(958)\nu_\tau) &< 4.0 \times 10^{-6}. \end{aligned}$$

The limits are determined from the weighted average of the branching fractions measured for each mode. The $\tau^- \rightarrow \pi^-\pi^0\eta'(958)\nu_\tau$ and $\tau^- \rightarrow K^-\eta'(958)\nu_\tau$ channels are potential backgrounds to the $\tau^- \rightarrow \pi^-\eta'(958)\nu_\tau$ decay. We find that background from these two decays is less than two events based on the upper limits on the branching fractions and we consider these backgrounds to be negligible. The previous limits on the $\tau^- \rightarrow \pi^-\eta'(958)\nu_\tau$ decay were measured by *BABAR* to be 7.2×10^{-6} [13] and by *CLEO* to be 8×10^{-5} [19]. It is predicted that the branching fraction of $\tau^- \rightarrow \pi^-\eta'(958)\nu_\tau$ should be less than 1.4×10^{-6} [27].

F. Searches for decays involving charged kaons

Finally we present the first search for high-multiplicity τ decays with one or two charged kaons. We find no evidence for signal decays and place upper limits on the branching fractions of the $\tau^- \rightarrow K^-2\pi^-2\pi^+\nu_\tau$, $\tau^- \rightarrow K^+3\pi^-\pi^+\nu_\tau$, $\tau^- \rightarrow K^-K^+2\pi^-\pi^+\nu_\tau$, $\tau^- \rightarrow K^-2\pi^-2\pi^+\pi^0\nu_\tau$, $\tau^- \rightarrow K^+3\pi^-\pi^+\pi^0\nu_\tau$, and $\tau^- \rightarrow K^-\eta'(958)\nu_\tau$ decay modes (the $\tau^- \rightarrow K^-\eta'(958)\nu_\tau$ decay was presented in Section E).

The events are divided into topologies in which the charged kaon has either the same or opposite charge as the parent τ lepton. If there are two kaon candidates, they must have opposite charge. All other tracks are required to be identified as charged pions. The selection criteria and systematic uncertainties are described earlier. The requirement on the invariant mass ($M < 1.8 \text{ GeV}/c^2$) of the final state uses the kaon mass for tracks identified as charged kaons. Figure 9 shows the mass spectra for the various channels. The predictions of the Monte Carlo simulation are divided into decays with or without a K^- (in this latter case, a π^- is misidentified as a K^-). The figures do not include any signal decays in the Monte Carlo samples. The background estimates, which give the dominant systematic uncertainty, are verified by comparing the numbers of events in the data and Monte

TABLE VI: Results and branching fractions for $\tau^- \rightarrow \pi^- \pi^0 \eta'(958) \nu_\tau$, $\tau^- \rightarrow K^- \eta'(958) \nu_\tau$, and $\tau^- \rightarrow \pi^- \eta'(958) \nu_\tau$ decays.

$\tau^- \rightarrow \pi^- \pi^0 \eta'(958) \nu_\tau$	$\eta \rightarrow \gamma\gamma$	$\eta \rightarrow \pi^+ \pi^- \pi^0$	
Limit (90% C.L.)	1.4×10^{-5}	1.9×10^{-5}	
Branching fraction (10^{-6})	$7.8 \pm 4.1 \pm 1.7$	$0.0 \pm 7.6 \pm 9.3$	
Data events	24 ± 10	5 ± 6	
Background events	5 ± 7	5 ± 8	
Selection efficiency	$(1.58 \pm 0.02)\%$	$(1.00 \pm 0.03)\%$	
$\tau^- \rightarrow K^- \eta'(958) \nu_\tau$	$\eta \rightarrow \gamma\gamma$	$\eta \rightarrow \pi^+ \pi^- \pi^0$	
Limit (90% C.L.)	2.4×10^{-6}	4.2×10^{-6}	
Branching fraction (10^{-6})	$0.5 \pm 1.3 \pm 0.4$	$1.6 \pm 1.4 \pm 1.2$	
Data events	6 ± 7	15 ± 4	
Background events	3 ± 4	11 ± 3	
Selection efficiency	$(3.47 \pm 0.03)\%$	$(3.09 \pm 0.04)\%$	
$\tau^- \rightarrow \pi^- \eta'(958) \nu_\tau$	$\eta \rightarrow \gamma\gamma$	$\eta \rightarrow \pi^+ \pi^- \pi^0$	$\eta \rightarrow 3\pi^0$
Limit (90% C.L.)	5.2×10^{-6}	9.0×10^{-6}	1.3×10^{-5}
Branching fraction (10^{-6})	$-2.8 \pm 3.5 \pm 1.9$	$-0.4 \pm 3.9 \pm 4.3$	$-1.8 \pm 8.1 \pm 3.3$
Data events	40 ± 22	44 ± 11	12 ± 10
Background events	58 ± 12	45 ± 12	14 ± 4
Selection Efficiency	$(4.06 \pm 0.35)\%$	$(3.25 \pm 0.15)\%$	$(0.96 \pm 0.10)\%$

Carlo samples in the $M > 1.8 \text{ GeV}/c^2$ region.

The numbers of events selected in the data and Monte Carlo simulations are given in Table VII. The backgrounds predicted by the Monte Carlo simulations are approximately equal to the numbers of events in the data sample. There is an excess of data events in the $\tau^- \rightarrow K^- 2\pi^- 2\pi^+ \pi^0 \nu_\tau$ mode, but this excess extends to mass values above the τ mass, indicating that events are due to background τ decays or $q\bar{q}$ events.

The upper limits on the branching fractions are given in Table VII. There are no predictions for these modes. We estimate that $\mathcal{B}(\tau^- \rightarrow K^- 2\pi^- 2\pi^+ \nu_\tau) \sim 10^{-5} - 10^{-6}$ if the decay is related to $\mathcal{B}(\tau^- \rightarrow 3\pi^- 2\pi^+ \nu_\tau)$ by the ratio of the CKM matrix elements (V_{us}/V_{ud}). The $\tau^- \rightarrow 3\pi^- 2\pi^+ \pi^0 \nu_\tau$ decay is dominated by decays to the narrow low-lying resonances and the branching fraction of decay modes created by replacing a π^- with K^- would be highly suppressed due to the limited phase space.

IV. SUMMARY

We present measurements of the branching fractions for τ lepton decays to three-prong and five-prong final states. The results are summarized in Table VIII. The branching fractions exclude contributions of the K_s^0 meson. The results are more precise than previous measurements and many decay modes are studied for the first

time.

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TABLE VII: Results and branching fractions for charged kaon decay modes.

Decay Mode	$\tau^- \rightarrow K^- 2\pi^- 2\pi^+ \nu_\tau$	$\tau^- \rightarrow K^+ 3\pi^- \pi^+ \nu_\tau$	$\tau^- \rightarrow K^- K^+ 2\pi^- \pi^+ \nu_\tau$
Limit (90% C.L.)	2.4×10^{-6}	5.0×10^{-6}	4.5×10^{-7}
Branching fraction (10^{-6})	$0.6 \pm 0.5 \pm 1.1$	$1.6 \pm 0.6 \pm 2.4$	$0.30 \pm 0.10 \pm 0.07$
Data events	1328 ± 36	1999 ± 45	32 ± 6
Background	1284 ± 72	1890 ± 163	15 ± 4
Selection Efficiency	$(7.9 \pm 0.1)\%$	$(7.9 \pm 0.1)\%$	$(6.7 \pm 0.1)\%$

	$\tau^- \rightarrow K^- 2\pi^- 2\pi^+ \pi^0 \nu_\tau$	$\tau^- \rightarrow K^+ 3\pi^- \pi^+ \pi^0 \nu_\tau$
Limit (90% C.L.)	1.9×10^{-6}	8×10^{-7}
Branching fraction (10^{-6})	$1.1 \pm 0.4 \pm 0.4$	$-0.6 \pm 0.5 \pm 0.7$
Data events	112 ± 11	154 ± 12
Background	84 ± 10	170 ± 16
Selection Efficiency	$(2.9 \pm 0.1)\%$	$(2.9 \pm 0.1)\%$

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TABLE VIII: Summary of branching fractions excluding contributions from $K_S^0 \rightarrow \pi^+\pi^-$.

Decay Mode	Branching fraction
Resonant decays	
$\tau^- \rightarrow 2\pi^-\pi^+\eta\nu_\tau$ (including f_1)	$(2.25 \pm 0.07 \pm 0.12) \times 10^{-4}$
$\tau^- \rightarrow 2\pi^-\pi^+\eta\nu_\tau$ (excluding f_1)	$(0.99 \pm 0.09 \pm 0.13) \times 10^{-4}$
$\tau^- \rightarrow \pi^-2\pi^0\eta\nu_\tau$ (including f_1)	$(2.01 \pm 0.34 \pm 0.22) \times 10^{-4}$
$\tau^- \rightarrow \pi^-f_1\nu_\tau$ via $f_1 \rightarrow 2\pi^+2\pi^-$	$(5.20 \pm 0.31 \pm 0.37) \times 10^{-5}$
$\tau^- \rightarrow \pi^-f_1\nu_\tau$ via $f_1 \rightarrow \pi^+\pi^-\eta$	$(1.26 \pm 0.06 \pm 0.06) \times 10^{-4}$
$\mathcal{B}(f_1 \rightarrow 2\pi^+2\pi^-)/\mathcal{B}(f_1 \rightarrow \pi\pi\eta)$	$0.28 \pm 0.02 \pm 0.02$
$\tau^- \rightarrow 2\pi^-\pi^+\omega\nu_\tau$	$(8.4 \pm 0.4 \pm 0.6) \times 10^{-5}$
$\tau^- \rightarrow \pi^-2\pi^0\omega\nu_\tau$	$(7.3 \pm 1.2 \pm 1.2) \times 10^{-5}$
Non-resonant decays	
$\tau^- \rightarrow 3\pi^-2\pi^+\nu_\tau$ (excluding ω, f_1)	$(7.68 \pm 0.04 \pm 0.40) \times 10^{-4}$
$\tau^- \rightarrow 2\pi^-\pi^+3\pi^0\nu_\tau$ (excluding η, ω, f_1)	$(1.0 \pm 0.8 \pm 3.0) \times 10^{-5}$
$\tau^- \rightarrow 2\pi^-\pi^+3\pi^0\nu_\tau$ (excluding η, f_1)	$(16.9 \pm 0.8 \pm 4.3) \times 10^{-5}$
$\tau^- \rightarrow 3\pi^-2\pi^+\pi^0\nu_\tau$ (excluding η, ω, f_1)	$(3.6 \pm 0.3 \pm 0.9) \times 10^{-5}$
$\tau^- \rightarrow 3\pi^-2\pi^+\pi^0\nu_\tau$ (excluding η, f_1)	$(1.11 \pm 0.04 \pm 0.09) \times 10^{-4}$
Inclusive decays (including η, ω, f_1)	
$\tau^- \rightarrow 2\pi^-\pi^+3\pi^0\nu_\tau$	$(2.07 \pm 0.18 \pm 0.37) \times 10^{-4}$
$\tau^- \rightarrow 3\pi^-2\pi^+\nu_\tau$ (excluding ω)	$(8.33 \pm 0.04 \pm 0.43) \times 10^{-4}$
$\tau^- \rightarrow 3\pi^-2\pi^+\pi^0\nu_\tau$	$(1.65 \pm 0.05 \pm 0.09) \times 10^{-4}$
$\eta'(958)$ decays (90% upper level confidence limit)	
$\tau^- \rightarrow \pi^-\pi^0\eta'(958)\nu_\tau$	$< 1.2 \times 10^{-5}$
$\tau^- \rightarrow K^-\eta'(958)\nu_\tau$	$< 2.4 \times 10^{-6}$
$\tau^- \rightarrow \pi^-\eta'(958)\nu_\tau$	$< 4.0 \times 10^{-6}$
Kaonic decays (90% upper level confidence limit)	
$\tau^- \rightarrow K^-2\pi^-2\pi^+\nu_\tau$	$< 2.4 \times 10^{-6}$
$\tau^- \rightarrow K^+3\pi^-\pi^+\nu_\tau$	$< 5.0 \times 10^{-6}$
$\tau^- \rightarrow K^-K^+2\pi^-\pi^+\nu_\tau$	$< 4.5 \times 10^{-7}$
$\tau^- \rightarrow K^-2\pi^-2\pi^+\pi^0\nu_\tau$	$< 1.9 \times 10^{-6}$
$\tau^- \rightarrow K^+3\pi^-\pi^+\pi^0\nu_\tau$	$< 8 \times 10^{-7}$